

The graph of function  $f$  is shown on the right.

The graph consists of a diagonal line, arcs of 2 circles, then another diagonal line.

SCORE: \_\_\_\_ / 4 PTS

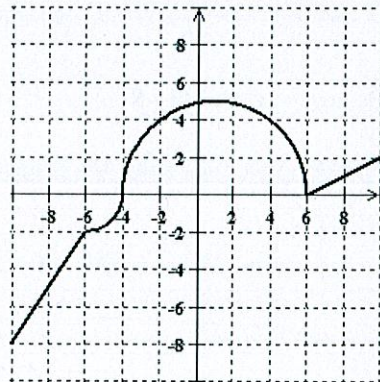
[a] Evaluate  $\int_{-10}^{10} f(x) dx$ .

**NOTE:** You must show the arithmetic expression that you used to get your answer.

$$\begin{aligned} & \left(\frac{1}{2}\right) \frac{1}{2} \pi (5)^2 + \frac{1}{2} 4 \cdot 2 - \frac{8+2}{2} \cdot 4 - \frac{1}{4} \pi (2)^2 \\ &= \frac{25}{2} \pi + 4 - 5 \cdot 4 - \pi \\ &= \frac{23}{2} \pi - 16 \end{aligned}$$

[b] Evaluate  $\int_6^{-10} f(x) dx$ .

$$-\int_{-10}^6 f(x) dx = -\left[\frac{25}{2} \pi - 5 \cdot 4 - \pi\right] = -\left(\frac{23\pi}{2} - 20\right) = 20 - \frac{23\pi}{2}$$



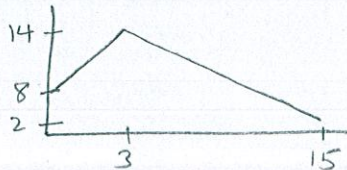
A person's velocity (in meters per second) at time  $t$  (in seconds) is given by  $v(t) = \begin{cases} 2t+8, & 0 \leq t \leq 3 \\ 17-t, & 3 \leq t \leq 15 \end{cases}$

SCORE: \_\_\_\_ / 5 PTS

- [a] Find the exact distance the person travelled from time  $t = 0$  seconds to  $t = 15$  seconds.

**NOTE: You must show the arithmetic expression that you used to get your answer.**

**You may only use techniques discussed in sections 5.1 and 5.2.**



$$\begin{aligned} & \left(\frac{1}{2}\right) \underbrace{\frac{8+14}{2} \cdot 3}_{11 \cdot 3} + \underbrace{\frac{14+2}{2} \cdot 12}_{8 \cdot 12} \left(\frac{1}{2}\right) \\ & = 11 \cdot 3 + 8 \cdot 12 \\ & = 33 + 96 = 129 \text{ METERS} \left(\frac{1}{2}\right) \end{aligned}$$

- [b] Estimate the distance the person travelled from time  $t = 0$  seconds to  $t = 15$  seconds using three subintervals and left endpoints.

**NOTE: You must show the arithmetic expression that you used to get your answer.**

$$\Delta t = \frac{15-0}{3} = 5$$



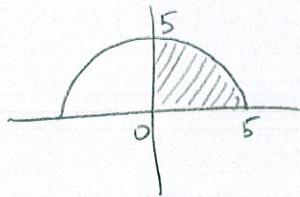
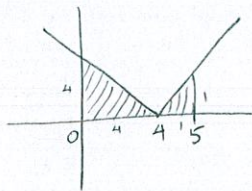
$$\begin{aligned} & v(0)\Delta t + v(5)\Delta t + v(10)\Delta t \\ & = 8 \cdot 5 + 12 \cdot 5 + 7 \cdot 5 \\ & = \left(\frac{1}{2}\right) \underbrace{(8+12+7)}_{27} \cdot \underbrace{5}_{5} \left(\frac{1}{2}\right) \\ & = 27 \cdot 5 = 135 \text{ METERS} \left(\frac{1}{2}\right) \end{aligned}$$



Evaluate  $\int_0^5 (|x-4| - 8\sqrt{25-x^2}) dx$  using the properties of definite integrals and interpreting in terms of area. SCORE: \_\_\_\_ / 5 PTS

**NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.**

$$\begin{aligned}
 \int_0^5 |x-4| dx - 8 \int_0^5 \sqrt{25-x^2} dx &= \frac{1}{2} \cdot 4 \cdot 4 + \frac{1}{2} \cdot 1 \cdot 1 - 8 \cdot \frac{1}{4} \pi (5)^2 \\
 &= \frac{17}{2} - 50\pi
 \end{aligned}$$



Using the limit definition of the definite integral, and right endpoints, find  $\int_{-2}^1 (2x^2 + 8x) dx$ .

SCORE: \_\_\_\_ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{1 - (-2)}{n} = \frac{3}{n}$$

$$\textcircled{1} \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-2 + \frac{3i}{n}\right) \frac{3}{n} \textcircled{1}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ 2\left(-2 + \frac{3i}{n}\right)^2 + 8\left(-2 + \frac{3i}{n}\right) \right] \textcircled{1/2}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ 2\left(4 - \frac{12i}{n} + \frac{9i^2}{n^2}\right) - 16 + \frac{24i}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ 8 - \frac{24i}{n} + \frac{18i^2}{n^2} - 16 + \frac{24i}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ -8 + \frac{18i^2}{n^2} \right] \textcircled{2}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left[ -8n + \frac{18}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \lim_{n \rightarrow \infty} 3 \left[ -8 + 3\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) \right] \textcircled{1}$$

$$= 3 \left[ -8 + 3 \cdot 1 \cdot 2 \right] \textcircled{1}$$

$$= 3(-2)$$

$$= -6 \textcircled{1} \text{ ONLY IF YOU USED THE } \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{ DEFINITION + METHOD}$$